

Analysis of the Thermal Stress and Strain on Arrigo Fiammingo's Artistic Window in the Cathedral of Perugia

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Thermal stress can damage fragile materials such as glass. It is a worrisome problem if the glass is a work of art, such as the polychromatic window of Arrigo Fiammingo (1565), in the Cathedral of Perugia, the topic of this paper. The window surface, irradiated by sunlight, suffers different thermal stresses, according to the color of the glass elements. In the present paper a calculation of stresses and strains on the window is carried out, for different temperature distributions due to sunlight, by using the ANSYS 5.3 software program. Results are compared with the actual state of the fissures on the window. [DOI: 10.1115/1.1404120]

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1 Introduction

Building materials and components are often exposed to thermal stress; in the long run it could be harmful. Effects on fragile materials such as glass could be significant, because they can break if subjected to large and rapid fluctuations in temperature. High thermal stresses are connected with direct solar radiation on the surfaces, more than with micro-climatic conditions, such as air temperature. Temperature fluctuations are rapid if induced by solar radiation more so than by equilibrium with the air temperature, which is a slower process. In a day, over a material with a certain exposure, incident solar radiation varies between 10–50 W/m² (diffuse radiation) to 100–400 W/m² (total radiation). Material temperature fluctuations could change by ten or more °C; the process could affect different parts of the component at the same time and not only the same part of the material at different times. When materials or components are works of art, such as the artistic windows in the churches of the Romanesque and Gothic age, the problem is rather important. These windows play a dominant role in natural lighting and iconographical decoration. The Umbria region in Italy, in particular, has many artistic windows: in the Cathedral of Perugia, in Saint Domenico's Church, one of the biggest in Europe and the wonderful windows of the Basilica of Saint Francis in Assisi are just examples.

The Thermotechnical and Environmental Control Laboratory of the Department of Industrial Engineering at the University of Perugia has been focusing for some time now on the conservation of works of art, with special attention on the microclimate (thermal and hygrometric conditions), lighting and air quality conditions [1].

The present paper was developed in connection with the intervention that the Superintendence of Environmental, Architectural, Artistic and Historical Works of Art of Umbria carried out on the just recently restored Arrigo Fiammingo window, located on the right aisle of the Cathedral of Perugia. The glass represents "The Sermon of Saint Bernardino"; the saint is on a podium and is preaching to people absorbed in thought. At the top of the window the monogram of Christ is surrounded by angels,

which compose an ornamental motif. People are intensely and brightly colored and stand out against the architectural light background.

During restoration, a protective window was installed externally to the artistic one, in order to limit the damage that atmospheric and polluting agents could induce. This installation caused changes in the microclimate, thoroughly investigated in previous papers [2,3,4,5,6].

In the present paper, the deterioration caused by thermal stress induced on the window surface due to climatic variations and to sunlight is examined. An analysis of the temperature distribution over the artistic glass surface has been carried out by means of a calculation software program (ANSYS 5.3), in order to assess the resulting stresses, which cause fissures on the window itself. During the 1988–1993 restoration performed by the Superintendence many fissures were detected and afterward repaired. In particular the "grisaille" has been fixed, then the glass surface has been cleaned and the fissured tesserae have been bonded; finally putty has been made.

ANSYS 5.3 also assessed the strains and the corresponding stresses occurred in the artistic glass. Results were compared with the admissible values for stresses on the glass and with the actual situation of the glass, by surveying the present fissures and the analysis of those already repaired in the recent restoration.

2 Arrigo Fiammingo's Glass Characteristics

Arrigo Fiammingo's window is a window with one panel with maximum dimensions of 1.8×7 m (surface area: about 11 m²). The protective glass is placed on the external countersinking part of the window, at about 18 cm from the artistic window, which has about the same dimensions. Both artistic and protective windows have northerly exposure (see Fig. 1).

The artistic window is made of polychromatic glass; the protective glass is made up of two-coupled glass sheets glued together with epoxy resin: one sheet is in conventional glass, the other is in Murano (Venice) blown glass.

The spectral coefficients of transparency and reflection for global solar radiation and for visible radiation of the two glass sheets have been measured by a spectrophotometer [6], and have been used in this work. The global coefficients of the artistic window, first used for the ventilation system design, were evaluated as the average among the values found for the different colors, weight with the respective surface area. Overall, the following were ob-

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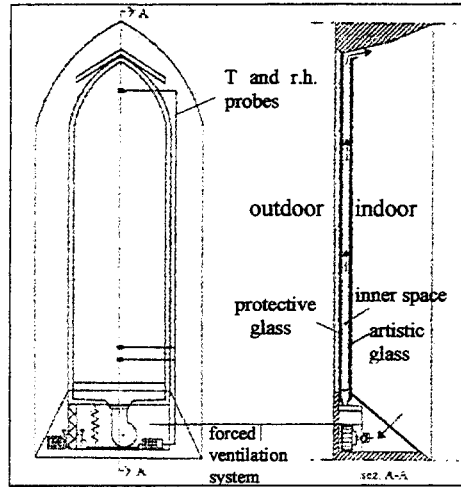


Fig. 1 Arrigo Fiammingo's window in the Cathedral of Perugia in its current state, with forced ventilation system in the space between artistic and protective glass

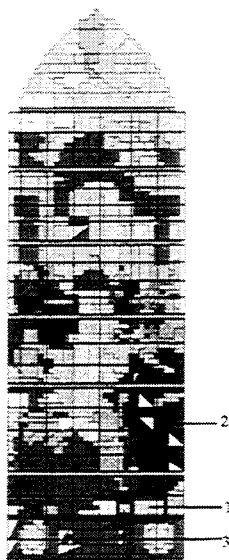


Fig. 2 Finite elements to analyze temperature, strain and stress distribution on Arrigo Fiammingo's glass; 1, 2, and 3 are the sections of the glass first examined

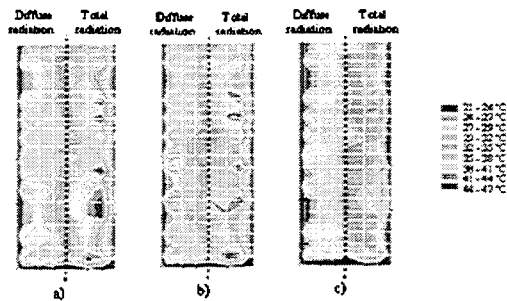


Fig. 3 Temperature distribution with direct and diffused sunlight (situation A): (a) without external protective glass; (b) with external protective glass and natural ventilation of the inner space; (c) with external protective glass and forced ventilation of the inner space.

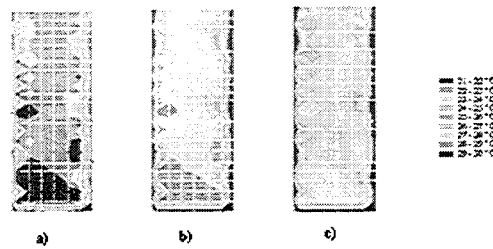


Fig. 4 Temperature distribution with only diffused sunlight (situation B): (a) without external protective glass; (b) with external protective glass and natural ventilation of the inner space; (c) with external protective glass and forced ventilation of the inner space.

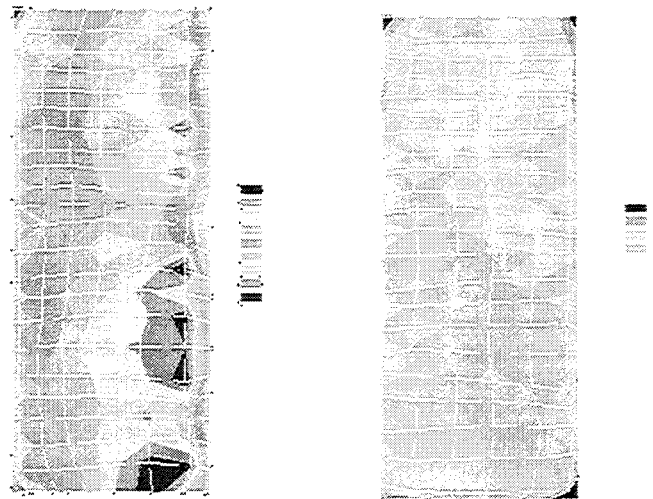


Fig. 9 Von Mises' stress trends on the glass in the situations (A) and (B), with protective glass and forced ventilation in the internal space.

tained: $t=0.03$, $r=0.10$; the corresponding absorption coefficient is $a=0.87$. The coefficients of transparency and reflection for visible radiation were $t=0.01$, $r=0.10$, and $a=0.89$. The protection glass has values $t=0.81$, $r=0.10$, and $a=0.09$; the coefficients of transparency and reflection for visible radiation were $t=0.83$, $r=0.10$, and $a=0.07$.

3 Thermal and Mechanical Properties of Different Glasses

The following thermal and mechanical properties of glass with different composition were employed in the calculation software ANSYS 5.3, to predict the fissures and the fiber cracks on Arrigo Fiammingo's window and to verify the actual fissures. In the final assessments, it must be considered that the final parameters found relate to new glass, whereas the artistic window was produced with different techniques many years ago. Its chemical composition—analyzed during the recent restoration process—revealed that it cannot be compared to a particular kind of glass, so data relating to silica glasses are considered [7]. The *coefficient of linear thermal dilation* α of glass varies according to the chemical composition; it is about 10^{-7} – 10^{-6} °C (see Table 1).

The *thermal conductivity of glass* does not vary much with temperature (between 0 and 100°C) and composition. Some values are reported in Table 1, but, on average, it is roughly 0.1 W/m°C. The *specific heat of the glass* composed of more than 60 percent of silica does not depend on the composition and is approximately 0.8 ± 0.1 kJ/kg°C at 25°C to 1.1 ± 0.1 kJ/kg°C at 500°C. Since glass is a fragile material and a poor conductor of heat, it doesn't withstand temperature variations. The temperature difference ΔT to which the object can be subjected rapidly without suffering damage is called *resistance to rapid temperature variations* (thermal shock, [8]). It depends on the thermal and mechanical properties of the material (traction resistance, Poisson's ratio, linear thermal dilation coefficient, Young's modulus, thermal conductivity, density and specific heat):

$$\Delta T = n \left[\frac{N(1-\mu)}{\alpha E} \right] \sqrt{\frac{\lambda}{\rho \gamma}} \quad (1)$$

The ΔT values for different glass thickness are reported in Table 2; they vary roughly in the range 35–180°C, for glass with different chemical composition and with thickness between 3 and 13 mm. Glass behaves like an elastic solid under transition temperatures; anywhere above this it becomes viscous. Its behavior in the glass transition interval is viscous-elastic. Therefore glass will not accept plastic or elastic-plastic deformation, but ruptures just as soon as it reaches its linear elastic limit. Glass follows Hooke's law and its modulus of elasticity depends on its composition, as reported in Table 3. The compressive resistance in glass is very good; traction resistance can't practically be measured due to micro-fissures which occur in the production phase. Rupture proceeds continuously once it is triggered, since it doesn't find propagation obstacles, as in critical materials (fragility). The kind of micro-fissures is purely random, therefore a sole value for mechanical resistance doesn't exist. So statistical measures are used to describe resistance distribution for a certain type of glass and Weibull's statistic distribution is used:

$$P = 1 - \exp[-(\sigma/\sigma_0)^m], \quad (2)$$

where P is the probability of rupture corresponding to the applied stress σ , σ_0 is a scalar parameter, and m is Weibull's modulus. Table 4 reports the safe loads (assessed with numerous trials) that different sized and thick sheets can withstand, with a safety factor of 2.5; they vary roughly in the range of 2–44 N/mm² for thickness between 3 and 12 mm and sheets with different sizes.

4 Temperature Distribution on the Artistic Glass

The assessment of the thermal strain and stress depends on the temperature distribution of the surface of the artistic glass; it is due to the microclimate and sunlight conditions. Some situations must be chosen in which thermal stress is more significant, so to attain the most critical events for glass strain and stress. To this aim, the temperature distribution is evaluated in the most critical situations, all concerning the month of July (see also [6]). July was chosen because it is characterized by the highest temperatures of the year and the section of the glass irradiated by sunlight is at its peak. In particular, there are four hours of sunlight in the morning and one in the afternoon; in the remaining part of the day the glass is in the shade. So two situations were chosen for sunlight (total and only diffused sunlight). The values for sunlight are reported in Table 5.

The examined situations are:

Table 1 Thermal properties of glass with different compositions versus temperature [8]

Glass	ρ (kg/m ³)	α_{0-100} (10 ⁻⁶ °C ⁻¹)	λ (W/m°C)	
			0 °C	100 °C
Silica glass	2200	0.5	0.132	0.148
Sodium-Calcium glass	2510	9.2	0.100	0.112
Boron-Silica glass	2230	3.3	0.112	0.129
Lead glass	2860	8.9	0.083	0.096
Aluminium-Silica glass	2520	4.2	0.137	0.129

Table 2 Resistance to rapid temperature variations for flat glass sheets for different composition and thickness [8]

Glass	ΔT (°C)		
	0.0032 m	0.0064 m	0.0126 m
Sodium-Calcium glass	60	50	35
Boron-Silica glass	180	150	100
Lead glass	65	50	35
Aluminium-Silica glass	135	115	75

Table 3 Young's Modulus E , tangent modulus G and Poisson's ratio μ for some glasses [8]

Glass	E (GPa)	G (GPa)	μ
Silica glass	73	31	0.16
Sodium-Calcium glass	72	29	0.25
Boron-Silica glass	64	27	0.20
Lead glass	61	25	0.21
Aluminium-Silica glass	88	35	0.25

Table 4 Safe loads in N/mm², which can bear on glass sheets of different dimensions and thickness [8]

Glass sheet dimensions (m)	Glass sheet thickness			
	0.003 m	0.007 m	0.010 m	0.012 m
1.83 x 1.83	14.6	21.0	29.8	43.9
1.12 x 3.05	13.2	19.0	27.3	39.5
1.83 x 2.43	1.7	15.6	22.4	33.2
1.83 x 3.05	8.8	12.7	18.1	26.4
2.43 x 3.05	5.9	9.3	13.7	20.0
1.83 x 4.27	5.9	8.8	13.2	19.0
3.05 x 3.05	5.3	7.3	10.7	16.1
2.43 x 4.27	4.9	6.8	9.8	14.2
3.05 x 4.27	3.9	5.4	7.8	10.7
3.05 x 6.1	2.4	3.9	5.4	7.8

Table 5 Hourly values of direct and diffused sunlight on the glass surface (W/m²) referred to the month of July [4]

Hour	Direct radiation (W/m ²)	Diffused radiation (W/m ²)	Total radiation (W/m ²)
	49.7	9.3	59.0
5	202.9	30.8	233.7
6	284.5	52.3	336.8
7	314.6	72.2	386.8
8	-	89.1	89.1
9	-	101.9	101.9
10	-	109.9	109.9
11	-	112.6	112.6
12	-	109.9	109.9
13	-	101.9	101.9
14	-	89.1	89.1
15	-	72.2	72.2
16	-	52.3	52.3
17	-	30.8	30.8
18	15.3	9.3	24.6
19			

A. direct and diffused sunlight on half of the artistic glass and only diffused sunlight on the other half, in the three following conditions:

- external protective glass not installed
- external protective glass installed, with natural ventilation in the internal space between the glasses
- external protective glass installed with forced ventilation in the internal space between the glasses

B. only diffused sunlight over all the glass, in the same three conditions.

Cases without the protective glass are particularly significant as they show the microclimatic history of the glass, in this situation from its installation in 1565 until 1993.

The most critical situations for the glass have been included among the examined ones. The ANSYS 5.3 software, based on finite elements, has been used for calculations. Hypotheses about finite elements of the system are described in [6]. The geometrical model was first built, composed of a net of rectangular elements (finite elements, see Fig. 2). The nodes of the net were matched as much as possible with the central points of the elements, so to distinguish the behavior of the glass with different colors and different absorption coefficients. The arched top of the window was left out of the analysis because the glass is in light color and the absorption coefficients are low [6]; so low temperatures and dilatations are expected.

The temperature variation between the nodes is considered linear; internal heating in each element is considered, equal to the absorbed solar energy (as a function of the absorption coefficient of the colored glass). The solar radiation absorption properties of the colored glasses used in this work have been measured by a spectrophotometer in a previous work [6].

A heat exchange between the nodes and the internal and external air is considered, so two grids are built: their points are respectively over an internal and an external plane parallel to the glass. In the nodes of the two grids, hypotheses on air temperature have been made:

- internal air: $T = \text{constant} = T_i$
- external air: with protective glass $T = \text{constant} = T_e$; without protective glass $T = \text{constant}$ for nodes at the same height: T increases from down to bottom with linear law (maximum growth equal to 2°C).

Discretization through the thickness was not considered because it is about 3 mm, so the glass dilatations in the thickness directions are negligible. The following environmental parameters have been assumed:

- adduction coefficient between air and artistic glass equal to $23.2 \text{ W/m}^2\text{C}$ without protective glass, equal to $8.14 \text{ W/m}^2\text{C}$ with protective glass and natural ventilation in the inner space, equal to $11.14 \text{ W/m}^2\text{C}$ with protective glass and forced ventilation in the inner space
- temperature around the glass is supposed equal to the wall and to the air inside the Cathedral temperature (about 21°C in July).

The following values of the thermophysical and mechanical properties of the glass have been chosen as input data:

- $\alpha = 5 \times 10^{-7} \text{ }^\circ\text{C}^{-1}$
- $\lambda = 0.136 \text{ W/m}^\circ\text{C}$
- $E = 73 \text{ GPa}$
- $G = 31 \text{ GPa}$
- $\mu = 0.16$

The results are shown in Figs. 3 and 4. As seen in [6], in the situation A, the maximum temperatures are found in the right half part of the glass, where there are both direct and diffused radiation; in that section, the highest temperatures are found in the

blue, red, and purple tesserae, with a higher absorption coefficient. The maximum temperatures range from 47°C when the protective glass is not installed (Fig. 3(a)) to 43°C with the protective glass and natural ventilation in the inner space (Fig. 3(b)), to 40°C with the protective glass and forced ventilation in the inner space (Fig. 3(c)). The minimum temperatures are found near the masonry and are around 21°C , so temperatures variations on the glass surface range, in the three conditions, from 26 to 19°C .

Figure 4 concerns situation B; it shows that temperature distribution depends only on the different colors of the tesserae, with higher temperatures in the darker colors of the glass (higher absorption coefficients). The maximum T value varies from 31 , to 30 , and to 28°C in the three mentioned conditions, with variations on the glass surface between 10 and 7°C .

On the bases of the temperature distribution on the situation A, corresponding to the most critical conditions for the glass, the strain and the stress were assessed; for sake of completeness, stress and strain were calculated on the situation B too.

5 Strain Analysis

An analysis of strain was performed considering the external protective glass and forced ventilation in the inner space, installed after restoration. Calculating the two extreme situations (this one and the one without protective glass), a maximum difference was detected for strain of $0.001 \mu\text{m}$. So it was thought more suitable to consider the present situation, with external protective glass and forced ventilation in the inner space, considering both situations A and B, to which the window is really subjected during the month of July.

The lead pieces that surround the glass tesserae create discontinuities on the surface, because they have different thermal properties from the glass, so they behave differently when temperature varies. First, instead of analyzing the whole glass, three sections only were examined (Fig. 2). The first (1) has some colors that contains lead pieces; the second (2) and the third (3) are respectively blue and yellow (maximum and minimum absorption coefficient values), don't contain lead pieces and lead only surrounds the section.

Each section of the glass chosen was divided in finite squared elements, considering the two different materials, glass and lead, and their different properties (Fig. 5). There aren't appreciable discontinuities between the dilation of glass and lead; results (Fig. 6) show that they differ slightly even in the most critical areas. So it could be hypothesized that the two materials do not interact that much: in fact the dilatation coefficients have the same magnitude (about $10^{-6} \text{ }^\circ\text{C}^{-1}$) and the presence of lead has not been considered in the analysis of the whole glass. The whole glass sheet was finally examined, neglecting the presence of lead and considering the window made up of only one material, the glass. Figs. 7(a) shows the trend of strain in the situation A; most of the dilatation is in the right hand side of the glass, where the highest temperatures due to sunrays are present. The maximum dilatation is $0.157 \mu\text{m}$ and it falls in the blue tesserae of the section under total radiation.

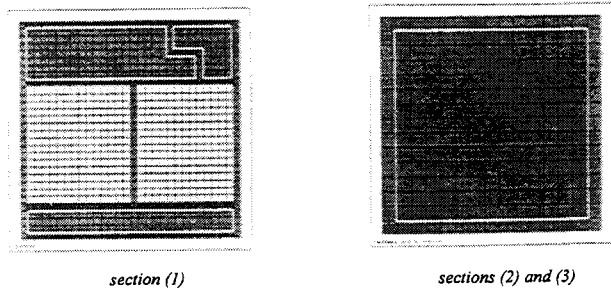
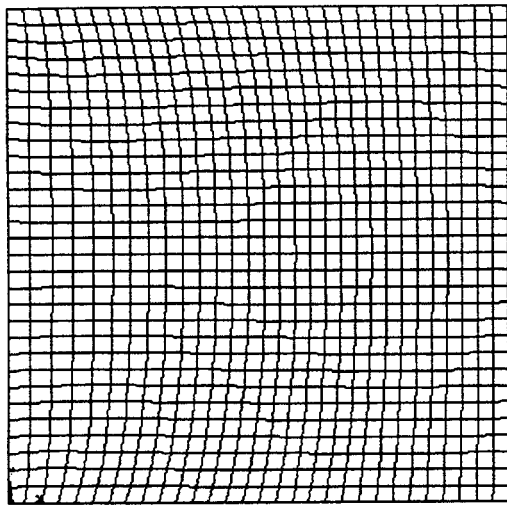
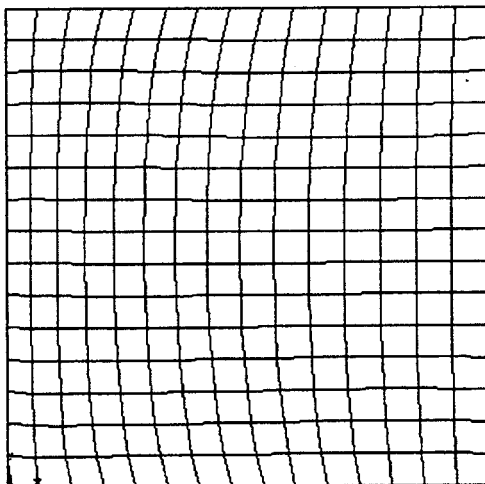


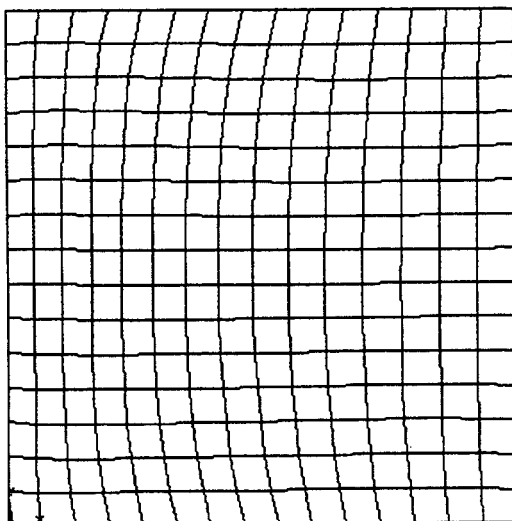
Fig. 5 Finite elements in glass sections (1), (2), (3)



section (1)

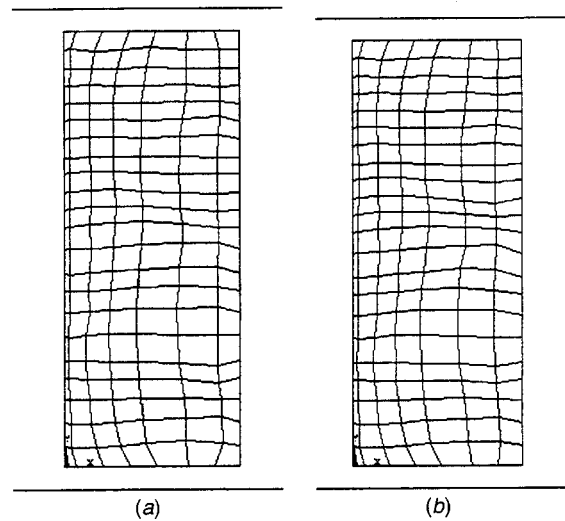


section (2)



section (3)

Fig. 6 Dilatation trends in glass sections (1), (2), (3)



(a)

(b)

Fig. 7 Dilatation trends on the artistic glass subjected to direct and diffused sunlight (situation A) and to only diffused sunlight (situation B)

Figure 7(b), concerning dilatations in the situation B, shows that they depend only on the color of the tesserae which take on higher values in the darker colors. The maximum dilatation is about $0.042 \mu\text{m}$, remarkably below those of the previous case (the scale in Fig. 7(b) is about 2.5 times the one in Fig. 7(a)).

A further verification about the lead influence was performed once the analysis was finished: a simulation was carried out considering the glass as if entirely made up of lead; the strain obtained was compared with those considering the window made up of only glass (see Fig. 8). The differences are negligible in all the portions of the glass sheet (about a few μm tenths in the worst conditions), thus confirming that the glass dilatation is not influenced by the surrounding lead.

6 Stress Analysis

Stress analysis was also performed, for the same reasons as in section 5, only in the conditions with external protective glass and forced ventilation in the space between the windows. We saw the maximum temperature variation from portion to portion of the glass was about 20°C . This value, although high, is lower than the limit value of the resistance to rupture variations of temperature, as indicated in Table 2. For glass sheets of 3 mm thickness the limit value is about 60°C .

The stress distribution that the temperatures induce on the glass was calculated by means of ANSYS software. The hypothesis is that the glass has fixed boundaries and it can't rotate at its edges. Stress values vary between 0.96 and 1 N/mm^2 in the situation A (Fig. 9(a)) and between 0.806 and 0.812 N/mm^2 in the situation B (Fig. 9(b)). They have been calculated in the different nodes with Von Mises method.

A comparison between calculated data and allowable stress limits has been carried out; allowable stress limits have been found in the Literature for glasses with chemical composition similar to the one of the Arrigo Fiammingo's window. The allowable stress value considered for the comparison is 2.4 N/mm^2 , corresponding to a 3 mm thickness glass, with $3.05 \times 6.10 \text{ m}$ dimensions (see Table 4). Artistic window's thickness is 3 mm, but its dimensions are about $1.8 \times 7 \text{ m}$, for a surface area smaller than the reference one. So we operate under safe acceptable conditions.

Maximum stress values calculated, respectively, 1 and 0.81 N/mm^2 in the situations A and B, are both lower than the stress allowable limit.

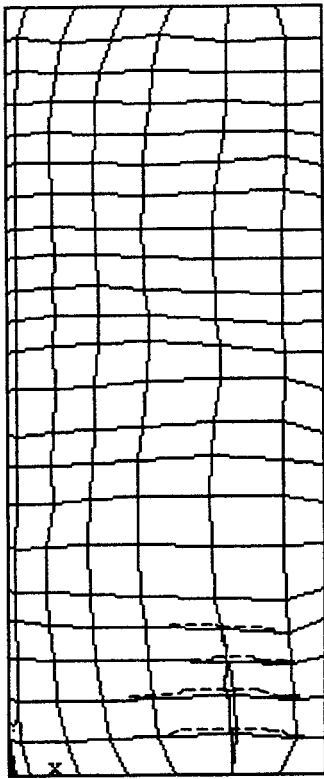


Fig. 8 Dilatation trend on the glass (situation A) when only lead is considered. the dashed lines show the dilatation differences of Fig. 7(a) (when only glass is considered).

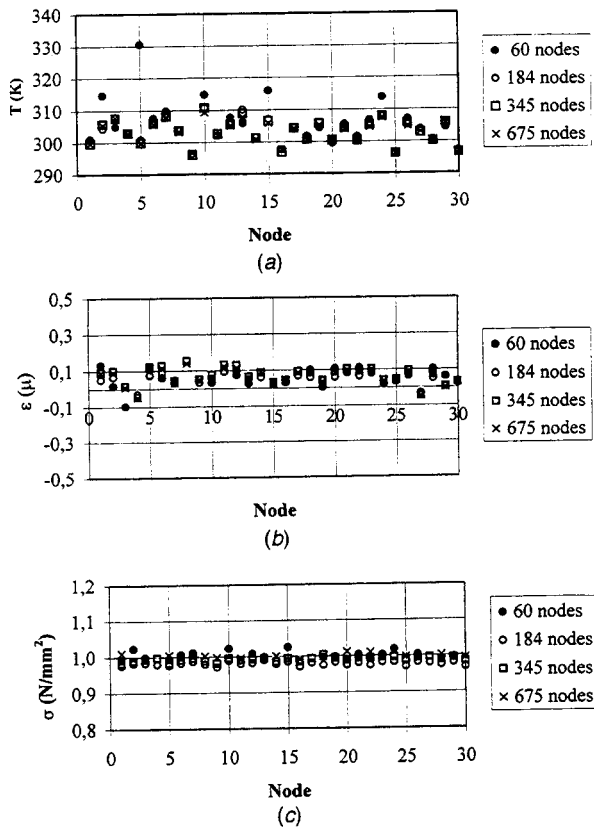


Fig. 10 Numerical accuracy: temperature (a), strain (b) and stress (c) values in 30 significant nodes, with different mesh refinements.

7 Numerical Accuracy

In order to evaluate the numerical accuracy of the model, four different mesh refinements were considered.

- 1 rectangular elements, with $5 \times 12 = 60$ nodes
- 2 rectangular elements, with $8 \times 23 = 184$ nodes
- 3 rectangular elements, with $15 \times 23 = 345$ nodes
- 4 rectangular elements, with $15 \times 45 = 675$ nodes

The temperature, strain and stress values in the different grid refinements were calculated and a comparison between the values of these parameters in 30 significant nodes was carried out.

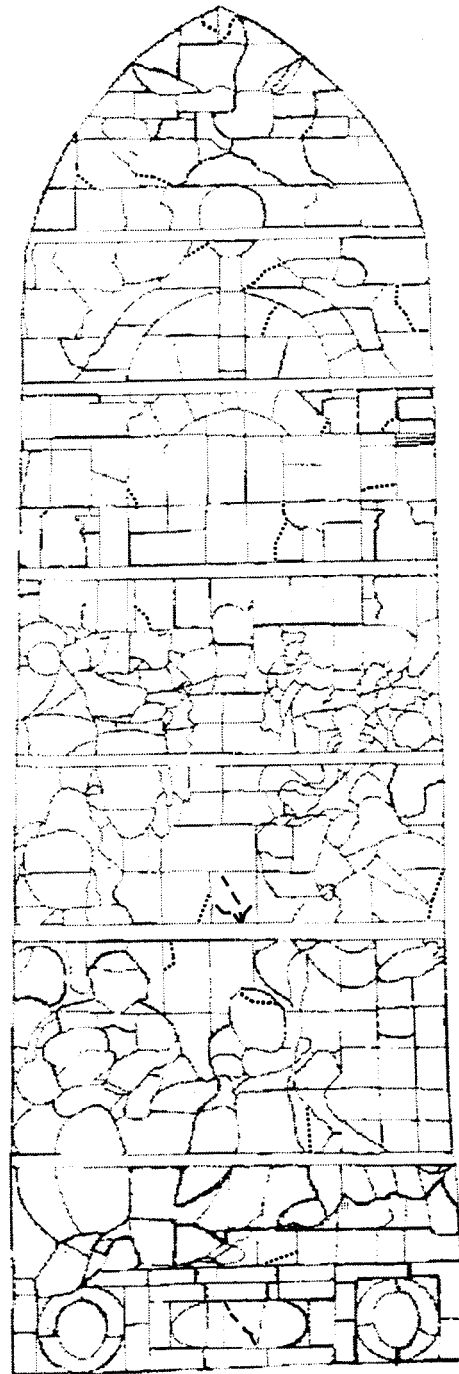


Fig. 11 Current lesions (dashed lines) and lesions repaired with added lead (dotted lines) during the 1998 restoration and before

Results are shown in Fig. 10(a,b,c), where temperature, strain and stress values in the 30 grid points chosen are reported, for the different mesh refinements; the mean difference between mesh 1 and 2 is significant (temperature differences are about $\pm(0-30)^{\circ}\text{C}$, strain differences are about $\pm(0.05-0.08)\ \mu\text{m}$ and stress differences are about $\pm(0.02-0.04)\ \text{N/mm}^2$). The mean differences between mesh refinements 2 and 3 and mesh refinements 3 and 4 are of the same entity and are less significant; they could be considered as the numerical accuracy of the model. So the mean values of the numerical accuracy are

- about $\pm 0.4^{\circ}\text{C}$ for temperature calculations
- about $\pm 0.02\ \mu\text{m}$ for strain calculations
- about $\pm 0.01\ \text{N/mm}^2$ for stress calculations.

The mesh refinement no. 2 is therefore the one chosen for the simulation.

8 Fissure State

A check at the Cathedral of Perugia was performed in order to detect possible fissures on the surface of Arrigo Fiammingo's glass.

Many interventions had been carried out during the 1989 restoration, when fissures were welded with lead. Strands of lead were inserted to repair and support the fragments of glass tesserae damaged throughout the years (the so-called *added lead*). Figure 11 indicates these interventions with dotted lines; dashed lines show the fissures now present on the glass. The actual fissures are in the area where the strain of the glass is highest (see Fig. 7). Therefore, even after restoration, the thermal stress could have caused the glass tesserae rupture, although the stress values are below the limits.

So in the present conditions there is a temperature, strain and stress distribution, which should not cause lesions on the surface of the artistic glass. But the investigation was conducted assuming that the glass is in perfect condition, without considering the deterioration or aging factors: these could decrease the mechanical and resistance characteristics of the glass tesserae. Other environmental factors, which can act alone or worse yet simultaneously, can speed up the process of deterioration (temperature and relative humidity of the air, air pollution, natural or artificial lighting conditions, etc.) [1,3]. This explains why some lesions on the glass were found in the area most subjected to stress and strain.

9 Conclusions

This paper illustrates the investigation of the thermal stress and strain on a particularly precious artistic glass (Arrigo Fiammingo 1565) located in the Cathedral of Perugia. Stresses are due to the microclimate created in the space between the glass examined and the external protective glass (installed during a recent restoration), but overall to sunlight.

A computer software program (ANSYS 5.3) has been employed to calculate the temperature distribution on the glass both in its present situation, with a protective glass and a ventilation system, and in the situation preceding the restoration intervention in 1988. Afterwards, on the bases of temperature distribution over the glass, dilatations and stresses were found.

The study of temperature distributions shows remarkable differences from one area to another of the glass, due to partially direct sunlight irradiation in summer months; these differences reach the maximum value of 20°C in the most critical conditions.

Finally, thermal dilatation and stress analysis show that values are below rupture limits defined for modern glass with composition similar to the window. However the results do not reflect wear and deterioration conditions that the glass has suffered in time; in spite of the restoration, they have certainly diminished thermal and mechanical characteristics of the glass. So a thorough examination of the window has revealed some lesions in areas most exposed to stress and strain.

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Nomenclature

- a = absorption coefficient (l)
- E = Young's modulus (GPa)
- G = tangent modulus (GPa)
- m = Weibull's modulus (l)
- n = dimensional constant in relation (1)
- N = traction resistance (N/mm^2)
- P = fracture probability (l)
- r = reflection coefficient (l)
- $r.h.$ = relative humidity (%)
- t = transparency coefficient (l)
- T = temperature ($^{\circ}\text{C}$, K)
- α = linear thermal dilation coefficient ($^{\circ}\text{C}^{-1}$, K^{-1})
- γ = specific heat (J/kgK)
- ΔT = resistance to rapid temperature variation ($^{\circ}\text{C}$, K)
- ϵ = strain (m, μm)
- λ = thermal conductivity (W/mK)
- μ = Poisson ratio (l)
- ρ = density (kg/m^3)
- σ = stress (N/mm^2)

Subscripts

- e = external
- i = internal

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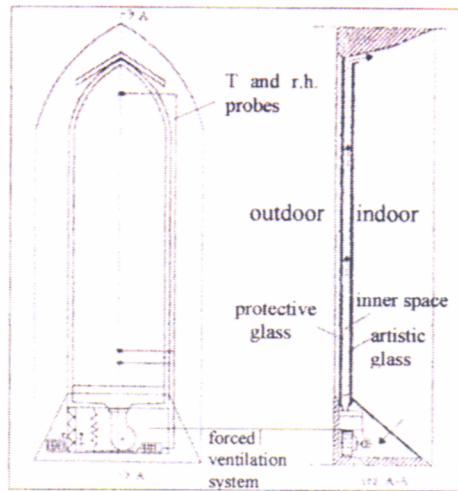


Fig. 1 Arrigo Fiammingo's window in the Cathedral of Perugia in its current state, with forced ventilation system in the space between artistic and protective glass

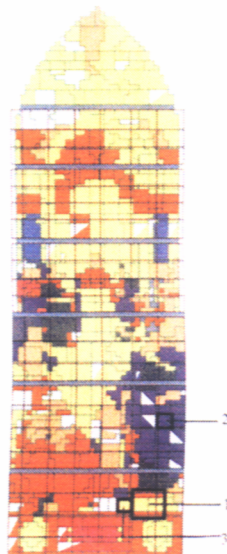


Fig. 2 Finite elements to analyze temperature, strain and stress distribution on Arrigo Fiammingo's glass: 1, 2, and 3 are the sections of the glass first examined

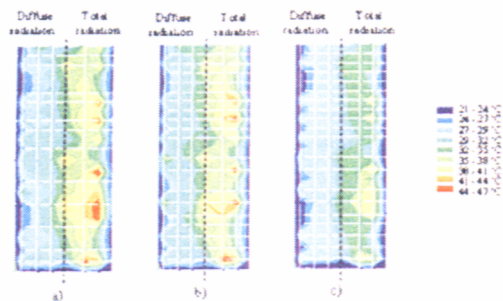


Fig. 3 Temperature distribution with direct and diffused sunlight (situation A): (a) without external protective glass; (b) with external protective glass and natural ventilation of the inner space; (c) with external protective glass and forced ventilation of the inner space.

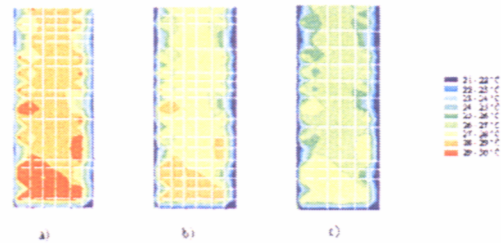


Fig. 4 Temperature distribution with only diffused sunlight (situation B): (a) without external protective glass; (b) with external protective glass and natural ventilation of the inner space; (c) with external protective glass and forced ventilation of the inner space.

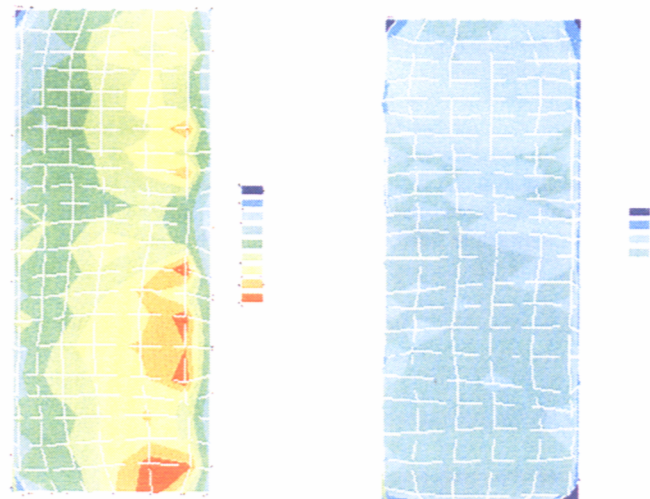


Fig. 9 Von Mises' stress trends on the glass in the situations (A) and (B), with protective glass and forced ventilation in the internal space.

tained: $t=0.03$, $r=0.10$; the corresponding absorption coefficient is $a=0.87$. The coefficients of transparency and reflection for visible radiation were $t=0.01$, $r=0.10$, and $a=0.89$. The protection glass has values $t=0.81$, $r=0.10$, and $a=0.09$; the coefficients of transparency and reflection for visible radiation were $t=0.83$, $r=0.10$, and $a=0.07$.