

# Modellisation of an active control system for exhaust emission

Federico Rossi<sup>a</sup>, Cinzia Buratti<sup>a</sup> and Andrea Nicolini<sup>a</sup>

<sup>a</sup> *Università degli Studi di Perugia, Dipartimento di Ingegneria Industriale, Laboratorio di Acustica, Via G. Duranti 1/A-4, 06125 Perugia, Italy*

## ABSTRACT

An original prototype for vehicle exhaust gas pipe noise reduction is proposed: the system is based on an active noise control technique. A loudspeaker generates the canceling noise which is generated by a DSP based numerical algorithm. Canceling noise signal is emitted inside a custom pipe which is coaxial to the final segment of vehicle exhaust duct; disturbing noise and canceling noise destructively interfere at the exhaust duct outlet section; outgoing noise is thus reduced. The theoretical investigation upon the acoustic performances of the prototype has been carried out by adopting the electrical equivalent circuit approach; each acoustic element has been associated to an equivalent electrical component: air volume canceling noise propagates through has been modeled by an equivalent electrical circuit. Exhaust gas duct behavior is associated to an electrical inductance. Prototype transfer function has been estimated by an electrical circuit simulation software. Acoustical performances have been improved by optimizing equivalent electrical parameters.

## I. INTRODUCTION

Noise emission due to engine exhaust gases is commonly contrasted by passive mufflers which dissipate acoustical energy by means of different methods [1].

Passive muffler efficiency is very low for low frequency noise components; furthermore, passive muffler introduces pressure drop which contributes to decrease global engine efficiency [2]. An original electronic muffler (EM) is proposed.

The advantages of EM with respect to the passive mufflers are the followings: 1) high efficiency for low frequency exhaust noise components; 2) very low pressure drop which determines an improvement of global engine efficiency. EM is based on an active noise control technique. It is equipped with a loudspeaker which generates the canceling noise; canceling noise and exhaust gas noise destructively interfere at the exhaust pipe outlet section. The result is a reduction of global exhaust noise emissions.

An active control system prototype for variable spin internal combustion engine exhaust noise reduction has been designed.

A modellisation of electronic muffler has been carried out in order to optimize EM efficiency.

Modellisation has been achieved by means of an equivalent electrical circuit (EEC) which associates each acoustical element to a corresponding electrical one.

Thus, an EEC may be tested on behalf of EM acoustical behavior. EEC components values have been varied in order to improve the exhaust noise abatement.

The results of the present study will support the final design and the realization of EM. EM prototype will be installed on a Fiat Marea SW 1900 JTD car, equipped with a Diesel engine.

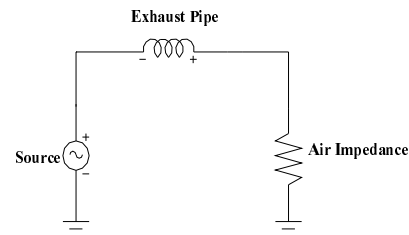
## II. ACOUSTIC WAVES IN EXHAUST PIPE

The proposed exhaust pipe is constituted by an empty tube; it is characterized by a constant circular section.

The exhaust pipe EEC is shown (fig. 1). An inductance corresponds to the empty tube. The inductance value is:

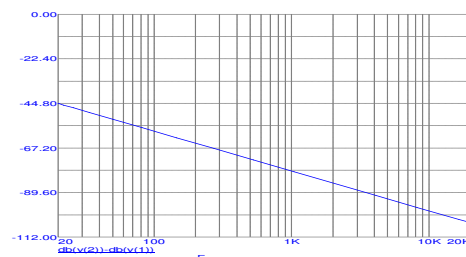
$$L = \frac{\rho \cdot l}{S} = 571.26 \left[ \frac{kg}{m^4} \right] \quad (1)$$

where  $l$  and  $S$  are respectively the exhaust pipe length and section area,  $\rho$  is the exhaust gas density.



**Figure 1:** equivalent electrical circuit of the exhaust pipe

The equivalent circuit is closed by the air impedance. The exhaust pipe EEC transfer function is shown (fig. 2). The graph shows that the exhaust pipe frequency response is a low pass type.



**Figure 2:** equivalent circuit transfer function of the exhaust pipe

### III. ELECTRONIC MUFFLER DESIGN

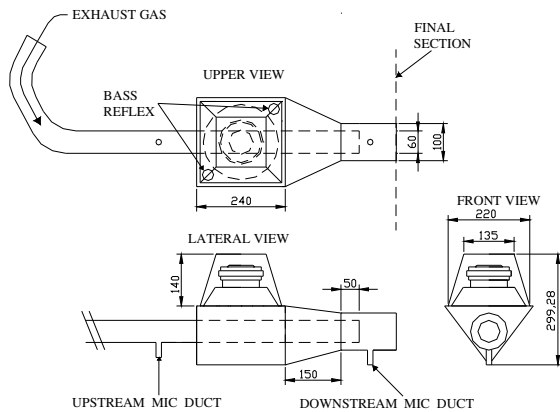
An active control noise muffler prototype has been designed. The prototype is specific for a four-stroke engine in order to be connected to a Fiat Marea JTD 105 exhaust pipe.

The purpose is to realize a destructive interference between the exhaust pipe noise signal and the canceling signal. The canceling signal is generated by a loudspeaker. The first part of the canceling noise pipe is characterized by a parallelepiped shape with triangular base. The exhaust pipe is a 3 mm thickness tube, in order to be acoustically insulated by the canceling noise duct.

The interference between the primary and the canceling signals occurs at the exhaust pipe outlet. Canceling noise duct final part has gradually been varied in order to obtain an impedance matching with external air. Canceling noise final section has a circular shape; it is coaxial to the exhaust pipe.

The loudspeaker is externally covered by a truncated pyramid shape acoustic box. The acoustic box is internally covered by absorbing material which concurs to improve the acoustical performances. The loudspeaker is fed, through an amplifier, by a control unit. The control unit implements a Filtered-x LMS control algorithm.

Electronic muffler design is showed in lateral, front and upper views (fig.3).



**Figure 3:** four-stroke engine electronic muffler design

Two 14 millimeters diameter holes (Bass Reflex) connect the acoustic box to canceling noise duct in order to increase the loudspeaker efficiency.

Two small tubes have been inserted upstream and downstream the electronic muffler. Microphones are inserted into the two tubes: the microphone placed upstream the electronic muffler permits to obtain a reference signal, the one placed downstream the muffler

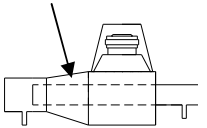
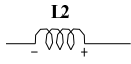
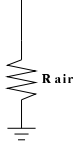
gives an error signal. A reference signal synchronized with the exhaust noise is obtained.

### IV. ELECTRONIC MUFFLER MODEL

The electronic muffler acoustic performances have been analyzed by an equivalent electrical circuit.

Each electrical circuit component has been associated to a specific muffler mechanical part (table 1).

Component	Muffler Component	Electrical Component
Acoustic Box		
Bass Reflex		
Canceling signal generated by the loudspeaker		
Canceling Noise Pipe		
Canceling Noise Pipe Outlet		

Electronic Muffler Outlet		
Air Impedance		

**Table 1:** analogies between muffler mechanical components and equivalent electrical components

The acoustic box is equivalent to a parallel RLC circuit [3].

The loudspeaker signal is modeled by a voltage generator  $E_g$ , while  $R_g$  and  $R_e$  represent respectively the generator internal resistor and the mobile coil electrical resistor.

Parallel RLC circuit values have been calculated using the following equations:

$$C_b = \frac{L_{ab} \cdot S_{Dm}^2}{B_m^2 \cdot l_c^2} = 5.8 \cdot 10^{-3} \left[ \frac{m^4 \cdot s^2}{kg} \right] \quad (2)$$

$$L_b = \frac{C_{ab} \cdot B_m \cdot l_c^2}{S_{Dm}^2} = 2.4 \cdot 10^{-4} \left[ \frac{kg}{m^4} \right] \quad (3)$$

$$R_b = \frac{B_m^2 \cdot l_c^2}{S_{Dm}^2 \cdot R_{ab}} = 1.9 \cdot 10^6 \left[ \frac{kg}{m^4 \cdot s} \right] \quad (4)$$

$R_{ab}$ ,  $C_{ab}$  e  $L_{ab}$  represent respectively acoustic box equivalent resistor, capacitor and inductance. They have been calculated as follows:

$$C_{ab} = \frac{V_{box}}{\gamma \cdot P_{box}} = 2.6 \cdot 10^{-7} \left[ \frac{m^4 \cdot s^2}{kg} \right] \quad (5)$$

$$L_{ab} = \frac{\rho \cdot l_{box}}{S_{box}} = 5.4 \left[ \frac{kg}{m^4} \right] \quad (6)$$

$$R_{ab} = \frac{\mu}{V_{box}} = 4.7 \cdot 10^{-4} \left[ \frac{kg}{m^4 \cdot s} \right] \quad (7)$$

$S_{Dm}$  is the loudspeaker diaphragm area;  $D_m$  and  $l_c$  are respectively magnetic induction and coil wire length.

The bass reflex is equivalent to a series RLC circuit [3]. Inductance, resistor and capacitor associated to a single hole have been calculated.

Later on, since the two holes can be considered equivalent to a parallel circuit, the following global equivalent component values have been calculated:

$$L_{h1} = L_{h2} = \frac{\rho}{D_{hm}} = 85.7 \left[ \frac{kg}{m^4} \right] \quad (8)$$

$$L_h = \frac{1}{\frac{1}{L_{h1}} + \frac{1}{L_{h2}}} = 42.8 \left[ \frac{kg}{m^4} \right]$$

$$R_{h1} = R_{h2} = \frac{\ln \frac{D_{hm}}{h_{hm}}}{4 \cdot S_{hm}} \cdot \sqrt{2 \cdot \rho \cdot \omega \cdot \mu} = 402.8 \left[ \frac{kg}{m^4 \cdot s} \right] \quad (9)$$

$$R_h = \frac{1}{\frac{1}{R_{h1}} + \frac{1}{R_{h2}}} = 201.4 \left[ \frac{kg}{m^4 \cdot s} \right]$$

$$C_{h1} = C_{h2} = \frac{V_{hm}}{\gamma \cdot P_{hm}} = 3.3 \cdot 10^{-12} \left[ \frac{m^4 \cdot s^2}{kg} \right] \quad (10)$$

$$C_h = C_{h1} + C_{h2} = 6.6 \cdot 10^{-12} \left[ \frac{m^4 \cdot s^2}{kg} \right]$$

$D_{hm}$ ,  $h_{hm}$ ,  $S_{hm}$  and  $V_{hm}$  are respectively the single hole diameter, thickness, section area and volume.

The first part of canceling noise path is a cavity; it is equivalent to a capacitor.

Canceling noise first part outlet corresponds to an inductance-resistor series circuit.

These equivalent electrical component values are calculated as follows [4]:

$$C1 = \frac{V_{cm}}{\gamma \cdot P_{cm}} = 2.8 \cdot 10^{-8} \left[ \frac{m^4 \cdot s^2}{kg} \right] \quad (11)$$

$$L1 = \frac{\rho}{D_{cm}} = 8.7 \left[ \frac{kg}{m^4} \right] \quad (12)$$

$$R1 = \frac{\ln \frac{D_{cm}}{h_{cm}}}{4 \cdot S_{cm}} \cdot \sqrt{2 \cdot \rho \cdot \omega \cdot \mu} = 9.5 \left[ \frac{kg}{m^4 \cdot s} \right] \quad (13)$$

$V_{cm}$  is cancelling noise duct volume;  $D_{cm}$ ,  $h_{cm}$  and  $S_{cm}$  are respectively cancelling noise pipe outlet section diameter, thickness and area.

Electronic muffler outlet pipe is equivalent to an inductance, calculated as a three elements sum: the first element corresponds to the variable section pipe length; the second element characterizes the constant section pipe length up to outlet exhaust pipe; the third element represents the external pipe extension beyond outlet exhaust pipe.

The inductance value is obtained as follows:

$$L2 = \frac{\rho}{\sqrt{M_{1m}}} \cdot \left( \frac{1}{\sqrt{S_{2m}}} - \frac{1}{\sqrt{S_{cm}}} \right) + \frac{\rho \cdot l_{2m}}{S_{2m}} + \frac{\rho \cdot l_{3m}}{S_{3m}} = 38.7 \left[ \frac{kg}{m^4} \right] \quad (14)$$

$M_{1m}$  is the variable section pipe slope,  $S_{2m}$  and  $l_{2m}$  are respectively the outlet pipe second segment section area and the length,  $S_{3m}$  and  $l_{3m}$  are respectively the outlet pipe third segment section area and the length. Finally, electronic muffler EEC (fig. 4) is closed by the air impedance.

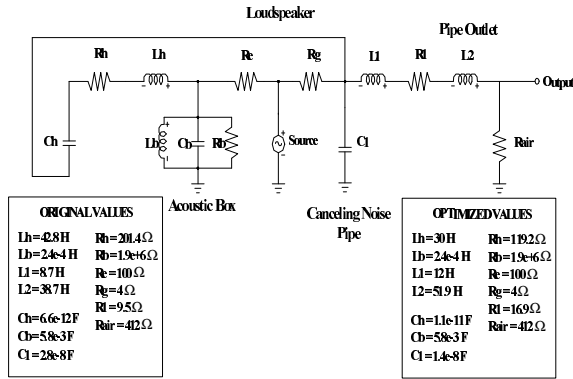


Figure 4: electronic muffler equivalent circuit

The transfer function of the EM equivalent circuit has been analyzed (fig. 5). It is equivalent to a low-pass filter in acoustic frequencies range.

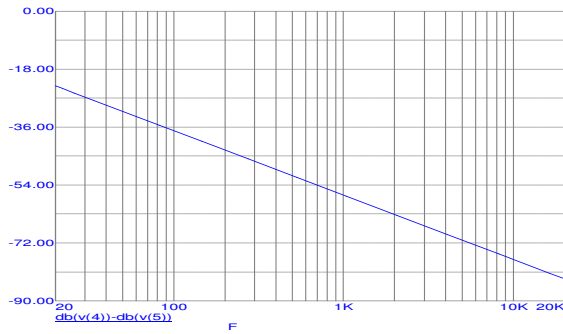


Figure 5: transfer function of the electronic muffler equivalent circuit

## V. MODEL OPTIMIZATION

The exhaust pipe model has been connected to the electronic muffler equivalent circuit. A square wave input signal is introduced into the exhaust duct model. Input signal spectrum is as close as

possible to the noise one. A canceling signal is obtained by a FxLMS algorithm. This signal is introduced into the electronic muffler model. The sum between the two signals is the error signal.

Muffler acoustical performances could be improved by carrying out the following adjustments: 1) decrease of canceling noise duct section area; 2) increase of bass reflex holes section area; 3) decrease of electronic muffler outlet pipe section area; 4) increase of acoustic box dimensions.

The adjustment 4) is not suitable because the optimized acoustic box dimensions would be too large to be contained into the assigned car box.

The optimized equivalent electrical components values are shown (fig. 4).

Table 2 shows the noise reduction improvement with respect to the noise reduction obtained with the original equivalent components values. Comparison has been lead introducing as noise signal a square wave signal with a 50-200 Hz fundamental component. It corresponds to a 1.5K-2K r.p.m. engine angular velocity range. The adjustments have introduced an approximately 3dB reduction for every engine angular velocity.

Square Wave Fundamental Frequency (Hz)	Optimized Reduction (dB)
50	3.3
75	2.7
100	3.6
125	3.5
150	2.6
175	3.1
200	3.3

Table 2: comparison between original and optimized values results

## VI. CONCLUSIONS

An original muffler prototype for noise reduction of vehicle exhaust gas pipe has been introduced; the muffler is based on an active noise control system. The theoretical investigation upon the prototype acoustics has been carried out by using electrical equivalent circuit approach; each mechanical element has been associated to an equivalent electrical component. The prototype transfer function has been estimated. The resulting transfer function is a low-pass type. Equivalent electrical model has been optimized by some adjustments. Better performances (about 3 dB) have been simulated; an experimental campaign aimed to prove theoretical result is actually going on.

## REFERENCES

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