ABSTRACT: Effect of fast electromagnetic transient on the thermal field in plane conductors is evaluated by means of the numerical simulation of the one-dimensional current and temperature distributions. It was found out that not negligible heating may occur along the conductor edge for electromagnetic pulse time constants lower than $10^{-3}$ s. Also, it is shown that heat exchange coefficient $h$ produces a negligible influence on temperature distribution. Finally, an analytical method to evaluate temperature distribution during short electromagnetic transients is given.

NOMENCLATURE

$\beta$ - electrical circuit time constant [s$^{-1}$]
d - conductor width [m]
$D$ - thermal diffusivity [m$^2$/s]
$D_e$ - Electrical diffusivity [m$^2$/s]
$E$ - Electrical field [N/Coulomb]
h - heat exchange rate [W/$^\circ$Km$^2$]
$H$ - Magnetic field [A/m]
i - imaginary unit
$I$ - global current [A]
$I_1$ - transient component of global current [A]
$I_2$ - harmonic component of global current [A]
$J$ - global current density [A/m$^2$]
$J_c$ - density of global current transient component [A/m$^2$]
$J_w$ - density of global current harmonic component [A/m$^2$]
$J^*$ - density of adimensional current
$l$ - thermal conductivity [W/$^\circ$Km]
$L^{-1}$ - inverse Laplace transform notation
$\varphi$ - initial phase of current harmonic component [rad]
$\mu$ - Magnetic permeability[Henry/m]
p - Laplace transform complex variable
$q$ - Laplace transform complex variable
1. INTRODUCTION

Magnetic Levitation, rail-guns systems and short circuit test on conductor bars are characterized by very high electrical power dissipation and very fast current transient [1], [2]. The present paper deals with a contemporaneous numerical evaluation of current and temperature transients on rectangular conductor. Evaluation has been carried out by means of a time dependent one dimensional finite differences model. The study allowed to determine the relations between current transient characteristics and temperature distribution; in fact, it has been shown that when current is higher than $10^5$ A and current rising time is shorter than $10^{-4}$ s, a sudden temperature increase occurs which may cause a local fusion of the conductor or, anyway, a degradation of the conductor mechanical properties [3],[4].

2. CURRENT DENSITY EVALUATION

Current versus time on rectangular conductors during short circuits has been experimentally determined which is given by the following equation:

$$I_1 = I_1 e^{-\beta t} + I_2 \cos(\omega t + \varphi).$$

$I_1, I_2$ and $\varphi$ satisfy the following constraints:

$$I(\tau = 0) = 0.$$  
Eq. (2) means:

$$\varphi = \arccos \left( -\frac{I_1}{I_2} \right).$$

In order to determine the current distribution inside a rectangular section conductor, the following hypotheses are assumed:

a) Electrical field is quasi-steady. Thus, drift current is neglected;

b) Electrical conductivity $\sigma$ is constant;

c) Conductor is considered an indefinite $d$ thick slab (see Fig.1). A reference system is assumed which shows $x$ direction parallel to slab $d$ thick section.

Hypotheses a) and b) allow to write the next equation [5]:

$$\vec{J} = \sigma \vec{E}.$$  
Thus magnetic field inside the conductor is given by:
\[ \Delta \vec{H} = \frac{1}{D_e} \frac{\partial \vec{H}}{\partial \tau}. \]  

(5)

\( D_e = \sigma \mu \) may be named electrical diffusivity by analogy to the thermal diffusivity.

Initial condition \((t=0)\) makes \( H=0 \) all over the conductor. Values of \( H \) on conductor surface depend on the current instantaneous value and the system geometry; thus they are known when eq. (1) is given. Current density is governed by the following relation:

\[ \vec{J} = \nabla \times \vec{H}. \]  

(6)

Hypothesis c) determines that current density is parallel to \( y \) direction and depends only on \( x \) coordinate and time (see Fig.1). Thus, according to (6), magnetic field direction is parallel to \( z \) axis (see Fig.1). Current density may be written as follow:

\[ J(x, \tau) = J_c + \text{Re}(J_\omega). \]  

(7)

Thanks to eq.(1) and eq.(6) we have:

\[ J_c = \frac{1}{2} I_1 \sqrt{\frac{\beta}{D_e}} \cos \left( x \sqrt{\frac{\beta}{D_e}} \right) e^{-\beta \tau} + 2 \frac{I_1}{d} \sum_{n=1}^{\infty} \frac{(-1)^n \cos \left( \frac{2 \pi n x}{d} \right)}{1 - \frac{\beta d^2}{4 \pi^2 n^2 D_e}} \cdot e^{-4 \pi^2 n^2 D_e \frac{\tau}{d^2}} \]  

(8)

and

\[ J_\omega = \frac{1}{2} I_2 (1 + i) \sqrt{\frac{\omega}{2 D_e}} \cosh \left[ x \left( 1 + i \right) \sqrt{\frac{\omega}{2 D_e}} \right] e^{i(\omega \tau + \varphi)} + 

+ 2 \frac{I_2}{d} e^{i \varphi} \sum_{n=1}^{\infty} (-1)^n \frac{\cos \left( \frac{2 \pi n x}{d} \right)}{1 + i \cdot \frac{\omega d^2}{4 \pi^2 n^2 D_e}} \cdot e^{-4 \pi^2 n^2 D_e \frac{\tau}{d^2}}. \]  

(9)

For common conductors \( D_e \equiv 10^{-2} \) is approximately a constant the order of magnitude of which is two times higher than thermal diffusivity: generally electromagnetic transients (see eq. (7), (8), (9) ) duration is much shorter than thermal transients.

Several cases of current distributions are reported in Fig. 2; they are shown in terms of the square value of the adimensional current which is defined below:

\[ J^* = \frac{d \cdot J}{I_2}. \]  

(10)

Thanks to symmetry, current distributions are given according to half space representation versus the following adimensional spatial variable:

\[ x^* = \frac{2x}{d}. \]  

(11)
Results (see Fig. 2) demonstrate that when the conductor maximum thickness is not much greater than $10^{-2}$ m, the magnetic field penetration time effects are already expired after $t=10^{-2}$ s. Furthermore, when $10^{-3}<\tau<10^{-5}$ s, the current transient cannot be neglected only if current rising time $T_s=1/\beta$ is short enough, that is to say $\beta>10^3$ s.

Fig. 2. Square values of adimensional current for different conditions.
3. TEMPERATURE TRANSIENT

Generally temperature field steady condition is reached much later than current field one which is represented by the first term of eqq. (8) and (9). Thus, temperature evaluation is achieved by keeping into account the heat source given by the steady terms of electrical power distribution which can be derived by the steady terms of current distribution. However, when current rising time is short enough, very high current values may locally and instantaneously occur; thus, heat dissipation may be locally and instantaneously very high; this phenomena happen very close to conductor surface (skin effect). Sometimes these facts may cause a quick temperature rise. In order to establish the relation between temperature and current transient conditions, a numerical evaluation has been carried out by implementing a FORTRAN based finite difference numerical simulation. The slab has been divided into a not equally spaced slices; each slice is modeled by a node the temperature of which is time dependent. Because of symmetry evaluations have been carried out only on half slab. The total number of nodes is 21 which are positioned as shown in Fig. 3.

Fig.3. Finite difference model: node positions.

The flux dissipated into the external environment has been kept into account by introducing a constant heat exchange rate. It was proved that different values of the heat exchange rate produce no meaningful differences of temperature distribution. Heat exchange rate values employed for simulations belong to 0 - 10 W/m²K range. Simulation results are reported in Fig. 4 which are shown in terms of the adimensional temperature defined below:

\[ \theta = \frac{T - T_a}{\Delta T} \]  

where \[ \Delta T = \frac{I_2^2}{4\sigma\lambda} \]  

(12)

I₂ represents the amplitude of the current harmonic component per unit of length. By analyzing the simulation results (see Fig. 4), if \( 10^{-3} \) s time interval is not elapsed since the beginning of current transient phenomenon, higher temperature values are found out near the slab edge rather than inside the slab. It may also be observed that temperature rapidly grows if current rise time diminishes. If regime current value would be considered, temperature values would have been higher than the values which have been found by considering a current transient; however, the simulated temperature values are much higher than the values which could have been found employing the same global current value with uniform distribution.
4. A THEORETICAL EQUATION FOR TEMPERATURE FIELD

Fig. 4 demonstrates that heat exchange rate does not influence temperature distribution at least when the elapsed time is shorter than 1 s. For such a condition a simplified theoretical form of temperature field may be introduced.

Assuming that environmental temperature is \( T_a = 0 \), let’s consider the following Fourier equation:

\[
\frac{1}{D} \frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial x^2} + \frac{J^2}{\sigma \lambda}. \tag{13}
\]
Eq. (13) constraints are:
\[ \tau = 0 \rightarrow T = 0 \]
\[ x = \frac{d}{2} \rightarrow \lambda \frac{\partial T}{\partial x} = -hT \quad e \quad x = -\frac{d}{2} \rightarrow \lambda \frac{\partial T}{\partial x} = hT. \]

Eq. (13) may be rewritten on Laplace variable domain:
\[
\frac{p}{D} \overline{T} = \frac{d^2 \overline{T}}{dx^2} + \frac{\overline{J}^2}{\sigma \lambda}. \tag{14}
\]

Eq. (14) constraints are:
\[ x = \frac{d}{2} \rightarrow \lambda \frac{\partial \overline{T}}{\partial x} = -h\overline{T} \quad e \quad x = -\frac{d}{2} \rightarrow \lambda \frac{\partial \overline{T}}{\partial x} = h\overline{T}. \]

Solution of eq.(14) may be written as follows:
\[ \overline{T} = \overline{T}_j + C \cdot \cosh(qx) \tag{15} \]
where:
\[ q = \sqrt{\frac{p}{D}} \tag{16} \]

\( \overline{T}_j \) is a particular solution of (13). Because of symmetry of \( \overline{J}^2 \) with respect to \( x \), the constant \( C \) of eq. (15) is given by the following relation:
\[
C = \frac{-h\overline{T}_j \left( \frac{d}{2} \right) - \lambda \left( \frac{d \overline{T}_j}{dx} \right)_{\frac{d}{2}}}{\lambda q \cdot \sinh \left( q \frac{d}{2} \right) + h \cdot \cosh \left( q \frac{d}{2} \right)}. \tag{17}
\]

The temperature distribution is governed by the following equation:
\[ T = \psi(x, \tau) + \int_0^5 F(x, t) \cdot \psi_1(\tau - t) \cdot dt \tag{18} \]
where \( \psi \) and \( F \) are the following inverse Laplace transform functions:
\[ \psi(x, \tau) = \mathcal{L}^{-1} \left( \overline{T}_j \right) \tag{19} \]
\[ F(x, \tau) = \mathcal{L}^{-1} \left( \frac{\cosh(qx)}{\lambda q \cdot \sinh \left( q \frac{d}{2} \right) + h \cdot \cosh \left( q \frac{d}{2} \right)} \right). \tag{20} \]

\( \psi_1 \) is the constraint defined below:
\[ \psi_1(\tau) = -h \cdot \psi \left( \frac{d}{2}, \tau \right) - \lambda \left( \frac{\partial \psi}{\partial x} \right)_{\frac{d}{2}} \tag{21} \]
Numerical results allow to suppose that \( h = 0 \); thus when \( d^2/d\tau \geq 1 \), eq. (1) assumes the following form [6]:

\[
F(x, \tau) = \frac{1}{\lambda} \sqrt{\frac{D}{\pi \tau}} \sum_{n=0}^{\infty} \left( e^{-\frac{(2n+1)^2}{4D\tau}} + e^{-\frac{(2n+1)^2}{4D\tau}} \right) \]

Eq. (22) describes the temperature distribution when current transient elapsed time is less than 1 second.

5. CONCLUSIONS

A numerical method was employed to evaluate the temperature distribution inside rectangular conductors when fast current transients occur. The investigation demonstrates that current transient influences temperature distribution only when current rising time is short enough (shorter than \( 10^{-3} \) s) which is represented by the inverse value of electrical circuit time constant. If the previous condition is verified, temperature may strongly rise near the conductor edge: maximum temperature value occurs at the same depth as magnetic field one. Temperature distribution is insensitive to both conductor dimensions if conductor is thicker than 0.01m and heat exchange rate when current transient elapsed time is shorter than 1s. Short circuit phenomenon of conductor bars are generally characterized by current rise time which does not determine sudden temperature rise; in such a cases temperature may be found by employing only current regime r.m.s. value. In many applications like rail guns or magnetic levitation, fast current transients may occur which last less than 1s and are characterized by rising time shorter than 0.1s; in such a cases transient current term must be kept into account to evaluate temperature distribution. A theoretical method was also proposed which describes early temperature distribution inside rectangular slab when quick current transient involve.

REFERENCES