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THEORETICAL APPROACH AND DESIGN OF AN ACTIVE VIBRATIONS CONTROL SYSTEM FOR TRANSPORTATION OF WORKS OF ART

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1. Introduction

Pieces of art are often removed from their natural place to be restored or shown in fairs, exhibitions and other main happenings; during transportation, damage may occur due to vibrations, which are usually contrasted by passive control systems.
In this paper, an active control system is presented, which produces vibrations of equal intensity as disturbing vibrations, but of opposite sign; thus the total effect is an elevated diminution of the total vibrations transmitted to the piece of art. The system is designed to control the range of frequency up to 10 Hz; higher frequencies vibrations, from 10 to 1000 Hz, are reduced by classical passive system as springs and dampers **[1]**.

2. Theoretical analysis

The theoretical study of an active control system to reduce vibration may be achieved by the scheme of Fig. 1.

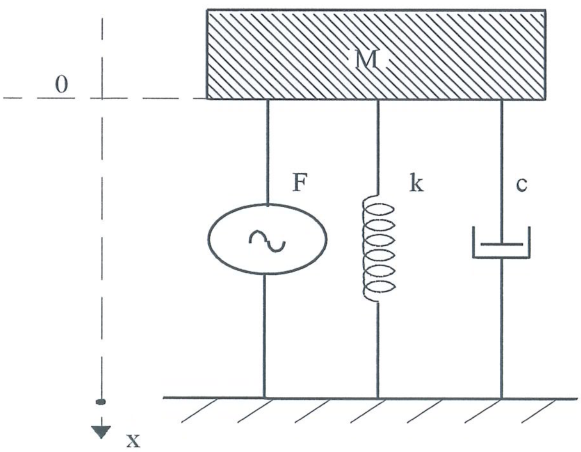

Fig.1: Physical reference scheme.

The physical behaviour of the system is described by a differential equation the solution of which supplies the mass displacement versus time.
With reference to Fig.1 it may be written:

$$m\ddot{x}+c\dot{x}+Kx = P - F \qquad (1)$$

Equation (1) is related to the vertical components of the vibration (Fig.1); other components are not taken into account.
If force "F" is absent and "P" is a step function, the solution of equation (1) is given by:

$$x(t)=\frac{P}{m\omega_n^2}+\frac{P}{m\omega_n^2}e^{-(c/c_c)\omega_n t}sin(\omega_d t-\theta_1) \tag{2}$$

where:

$$\omega_n=\sqrt{K/m} \quad ; \quad \omega_d=\omega_n\sqrt{1-(c/c_c)^2}$$

$$c_c=2m\omega_n \quad ; \quad \theta_1=atn\frac{\sqrt{1-(c/c_c)^2}}{-c/c_c} \tag{3}$$

With reference to equation (2), when "t" assumes increasing values, mass "m" reaches a new position defined by value "$P/(m\,\omega_n^2)$"; this value may be led to zero by introducing a control force "*F*" **[2]**.

Force "*F*" is proportional to the integral of the displacement, thus equation (1) may be rewritten as follows:

$$m\ddot{x}+c\dot{x}+Kx+G\int x\cdot dt=P \tag{3}$$

"G" represents the feedback gain.

The solution of equation (3) may be found by means of Laplace transform:

$$m\cdot s^2\cdot X(s)+c\cdot s\cdot X(s)+K\cdot X(s)+G\cdot\frac{X(s)}{s}=P \tag{4}$$

Solving equation (4) with respect to "X(s)":

$$X(s)=\frac{P}{m\cdot(s+q\omega_p)\cdot(s^2+2p\omega_p+\omega_p)} \tag{5}$$

the parameters "p", "q" e "ω_p" are defined by the relations:

$$\frac{c}{c_c}=\frac{2p+q}{2(1+2pq)^{1/2}}$$
$$\frac{G}{m\omega_n^3}=\frac{q}{(1+2pq)^{1/2}} \tag{6}$$
$$\omega_n=\omega_p(1+2pq)^{1/2}$$

The time-domain form of equation (5) is:

$$x=A\cdot e^{-q\omega_p\cdot t}+B\cdot e^{-p\omega_p\cdot t}sin(\omega_d t-\theta_2) \tag{7}$$

where:

$$\theta_2=a\,tan\frac{\sqrt{1-p^2}}{q-p} \tag{8}$$

The integrative control force may cause system instability; particular values of "G" determine the system self-oscillation even when the exciting force is absent.
System stability is attained when the following condition is satisfied **[3]**:

$$G \leq 2 \cdot \frac{c}{c_c} \cdot m \cdot \omega_n^3 \qquad (9)$$

3. The Active Control System

The system proposed in Fig.2 is a possible realisation of the scheme in Fig.1; it is made up of the following components:
1) A vibration damper device, which consists of two iron plates separated by four springs installed at the corners of the plates. The lower plate must be set on the ground where vibrations are produced; the upper plate is the base on which the work of art lies. The sum of the four springs is equivalent to the schematic spring of Fig.1; the sum of the upper plate and the work of art masses is the mass "m" relative to Fig.1 **[4]**.
2) An electronic control device which produces the feed signal for the actuator; it processes the signal coming from the sensor. The control transducer (actuator) is attached at the centres of the plates; it induces the controlled vibrations to the upper plate **[5]**. A sensor is placed on the lower plate surface, that generates a voltage signal proportional to the amplitude of vibrations the plate is excited with **[6]**. The friction of the mechanical components of the actuator are represented in Fig.1 as a viscous force with constant "c".

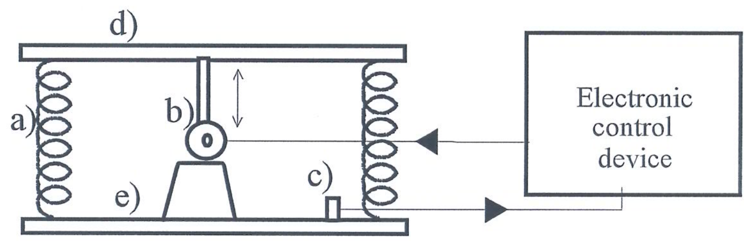

Fig.2: Scheme of the active control system:
a) springs; b) actuator; c) vibration sensor; d) upper plate; e) lower plate.

4. System performances evaluation

System performances are evaluated by the following data:
1) Maximum weight of work of art plus upper plate, m=25 Kg;
2) Natural frequency of the system, f_n=12 Hz;
3) Springs elastic constant, K=14.8 10^4 [N/m];
4) Viscous friction constant, c=400 [N s/m].
Equation (9) gives the maximum value of the feedback gain, $G=9 \cdot 10^6$. Supposing p=0.8 and q=0.2 and solving equation (6), $G= 1.7 \cdot 10^6$ may be found; this value satisfies the stability condition; furthermore, when p=0.8 and q=0.2, the vibrations transmitted to the work of art

are greatly attenuated; Fig.3 shows the system response to a step function in the cases of active control ON. and OFF.

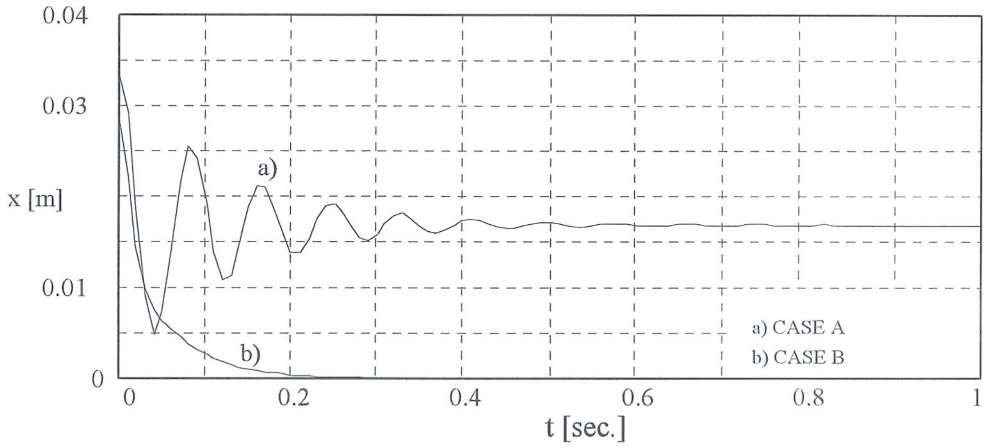

Fig 3: Time response for a step exciting force

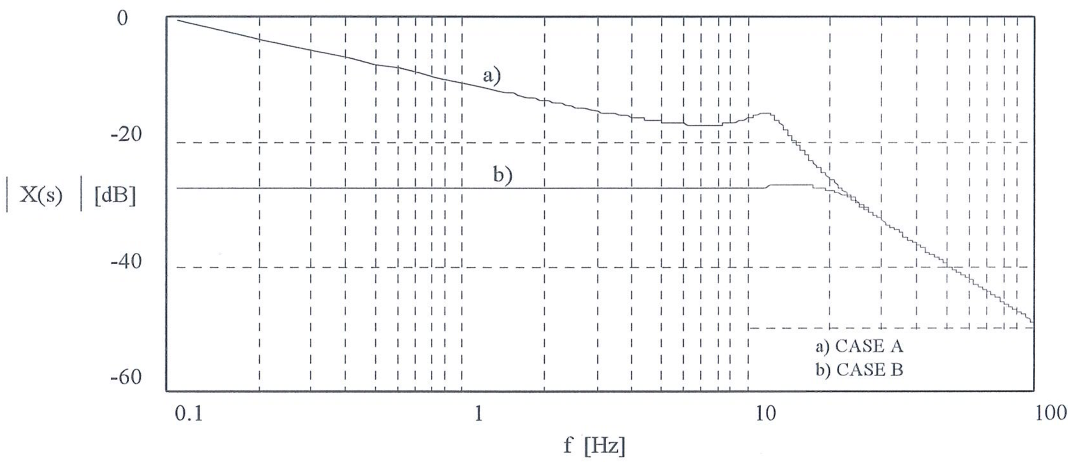

Fig 4: Frequency response for a step exciting force

If G=0 (q=0 and $p=c/c_c$) equation (5) supplies the system frequency response in absence of the control force (case A); such a response is compared in Fig.4 with the system response when $G= 1.7\cdot10^6$, q=0.2 and p=0.8 (case B); it may be observed that when the frequency is lower than 12 Hz (natural frequency), the attenuation produced by the system is much greater for case B than for case A; the value of attenuation, for case B, is about -25dB. When the value of frequency is greater than the natural frequency, the attenuation for case A is the same as B.

Furthermore, the logarithmic ratio "R" between the system response for cases A and B has been computed:

$$R = 10 \cdot \log \frac{|X_A(s)|}{|X_B(s)|} \qquad (10)$$

In Fig.5 "R" versus frequency is shown.

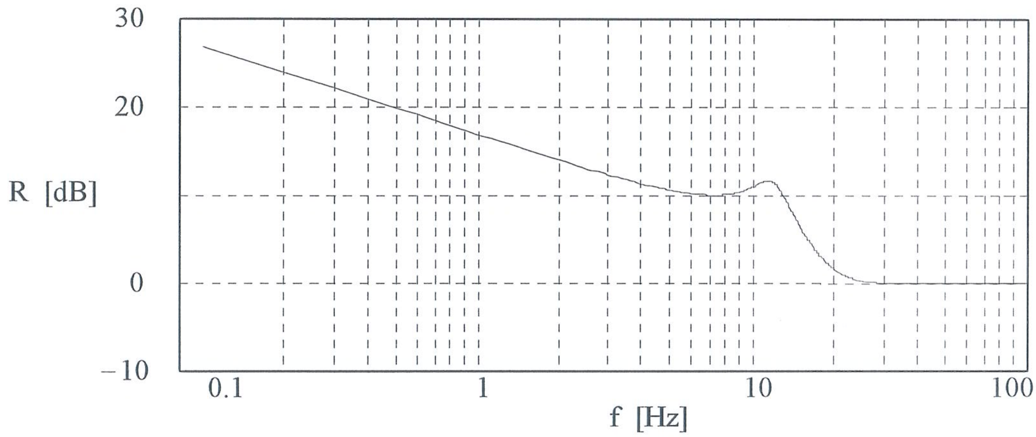

Fig.5: Ratio between the response of the system when the control is ON and when the control is OFF.

6. Conclusions

The traditional passive system used to insulate works of art against vibrations show good performances only for frequencies higher than the system natural frequency. This paper gives a theoretical approach to a new system able to reduce the lower frequencies, which are usually the main causes of damage **[7]**. The contemporary use of the proposed and traditional systems may reduce vibrations in the whole range of frequencies. The proposed system is based on an active control technique and may produce decrease in vibrations amplitude equal to 25dB in the range 1-12Hz. A prototype of the system is beeing developed at the Acoustic Laboratory of the University of Perugia.

7. List of Symbols

Letters

m= total mass [Kg];
K= total elastic constant [N/m];
c= viscous friction constant [N s/m];
P= exciting step force [N];
F= control force [N];
x= displacement [m];
f=frequency [Hz];
ω= pulsation [rad/s]
G= feedback gain [Kg/s^3];
s= complex variable for Laplace transform;
X= dispalcement on Laplace-domain.

Pedices

n=natural;
d=damped;
p=poles
A= case A;
B= case B.

8. Bibliography

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